Pumping lemma cheat sheet

For Theory of Computation / Automata Theory
THE 5-STEP PUMPING LEMMA TEMPLATE

STEP 1: Assume L is regular.

Let p be the pumping length from the pumping lemma.
Write: "Assume L is regular. Let p be the pumping length."

STEP 2: Choose a string $s \in L$ with $|s| \geq p$.

The string must depend on p.
Write: "Choose s = ___________ (|s| = ___ ≥ p)"

STEP 3: Consider ANY split s = xyz with $|xy| \leq p$ and $|y| \geq 1$.

Use the condition $|xy| \leq p$ to determine where y can be.
Write: "For any split s = xyz with $|xy| \leq p$ and $|y| \geq 1$..."

STEP 4: Show that $xy^0z \notin L$ OR $xy^2z \notin L$.

Pumping down (i=0) or up (i=2) creates a contradiction.
Write: "Then xy^0z = ___________ $\notin L$ (or $xy^2z \notin L$)"

STEP 5: Contradiction → L is NOT regular.

The pumping lemma requires $xy^iz \in L$ for all $i \geq 0$.
Write: "This contradicts the pumping lemma. Therefore, L is not regular."

KEY VARIABLES TO REMEMBER

p = pumping length (given by lemma)
s = chosen string (must depend on p)
x = prefix of s ($|xy| \leq p$)
y = middle part (non-empty, $|y| \geq 1$)
z = suffix of s
i = pumping factor (i = 0 or i = 2 for contradiction)

Solved Examples

SOLVED EXAMPLE 1: $\{0^n1^n \mid n \geq 0\}$

Problem: Prove L = $\{0^n1^n \mid n \geq 0\}$ is not regular.
Solution:
Step 1: Assume L is regular. Let p be the pumping length.
Step 2: Choose s = 0^p1^p. Then |s| = 2p ≥ p and s ∈ L.
Step 3: Consider any split s = xyz with |xy| ≤ p and |y| ≥ 1.
Since |xy| ≤ p, xy contains only 0s. So y = 0^k for some k ≥ 1.
Step 4: Pump down (i = 0): $xy^0z = 0^{p-k}1^p$.
This string has fewer 0s than 1s, so it is NOT in L.
Step 5: This contradicts the pumping lemma. Therefore, L is not regular.

SOLVED EXAMPLE 2: $\{a^nb^nc^n \mid n \geq 0\}$

Problem: Prove L = $\{a^nb^nc^n \mid n \geq 0\}$ is not regular.
Solution:
Step 1: Assume L is regular. Let p be the pumping length.
Step 2: Choose s = $a^pb^pc^p$. Then |s| = 3p ≥ p and s ∈ L.
Step 3: Consider any split s = xyz with |xy| ≤ p and |y| ≥ 1.
Since |xy| ≤ p, xy contains only a's. So y = a^k for some k ≥ 1.
Step 4: Pump down (i = 0): $xy^0z = a^{p-k}b^pc^p$.
This string has fewer a's than b's or c's, so it is NOT in L.
Step 5: This contradicts the pumping lemma. Therefore, L is not regular.

SOLVED EXAMPLE 3: {ww | w ∈ {a,b}*}

Problem: Prove L = {ww | w ∈ {a,b}*} is not regular.
Solution:
Step 1: Assume L is regular. Let p be the pumping length.
Step 2: Choose s = a^pba^pb. Then |s| = 2p+2 ≥ p and s ∈ L.
Step 3: Consider any split s = xyz with |xy| ≤ p and |y| ≥ 1.
Since |xy| ≤ p, y contains only a's from the first group.
Step 4: Pumping changes the number of a's in the first group only.
The resulting string cannot be of the form ww (unbalanced).
So $xy^iz \notin L$ for i ≠ 1.
Step 5: This contradicts the pumping lemma. Therefore, L is not regular.

COMMON MISTAKES TO AVOID

❌ MISTAKE 1: Choosing s that does NOT depend on p
WRONG: $s = 0^n1^n$
RIGHT: $s = 0^p1^p$
Why: The string must be expressed in terms of the pumping length p.

❌ MISTAKE 2: Assuming y is in a specific position
WRONG: "Let $y = 0^k$" without justification
RIGHT: "Since $|xy| \leq p$, xy contains only 0s, so $y = 0^k$"
Why: You must use the $|xy| \leq p$ condition to justify where y can be.

❌ MISTAKE 3: Forgetting the $|xy| \leq p$ condition
WRONG: Ignoring this condition entirely
RIGHT: "Because $|xy| \leq p$, we know that..."
Why: This condition is critical for determining the location of y.

❌ MISTAKE 4: Only checking i = 2 (pumping up)
WRONG: Only showing $xy^2z \notin L$
RIGHT: Consider i = 0 (pumping down) when it creates a cleaner proof
Why: Sometimes pumping down creates a simpler contradiction.

❌ MISTAKE 5: Choosing a string that doesn't allow contradiction
WRONG: Choosing $s = (ab)^p$ for language {ww}
RIGHT: Choosing $s = a^pba^pb$ for language {ww}
Why: Your chosen string must make the contradiction easy to show.

❌ MISTAKE 6: Not handling ALL possible splits
WRONG: "Assume $y = 0^k$"
RIGHT: "For any split with $|xy| \leq p$, y must consist only of 0s"
Why: The lemma says "for ANY split" — you must consider all possibilities.

PRACTICE PROBLEMS

Try to prove these languages are NOT regular using the pumping lemma.

PROBLEM 1: L = $\{a^n b^m \mid n \leq m\}$ (a's followed by b's, with fewer or equal a's)
PROBLEM 2: L = $\{0^i 1^j \mid i \neq j\}$ (different number of 0s and 1s)
PROBLEM 3: L = $\{a^{b^2} \mid b \geq 0\}$ (a raised to a perfect square power)
PROBLEM 4: L = $\{uuv \mid u, v \in \{a,b\}^*\}$ (strings with two identical prefixes)
PROBLEM 5: L = $\{a^n b^m \mid n = 2m\}$ (twice as many a's as b's)

ANSWER KEY (HINTS)

PROBLEM 1: Choose s = $a^p b^p$. Since $|xy| \leq p$, y contains only a's.
Pump down → fewer a's, still satisfies $n \leq m$? No, contradiction.
PROBLEM 2: Choose s = $0^p 1^{p+1}$. Pumping changes count of 0s only.
Resulting string has equal counts → contradiction.
PROBLEM 3: Choose s = a^{p^2}. Pumping changes length to $p^2 + k$.
Next perfect square is $p^2 + 2p + 1$, so result not a perfect square.
PROBLEM 4: Choose s = $a^p b a^p b$. Pumping changes first group only.
String no longer has form uuv.
PROBLEM 5: Choose s = $a^{2p} b^p$. With $|xy| \leq p$, y contains only a's.
Pumping changes a count, no longer twice b count.

EXAM TIPS

- Always start with "Assume L is regular" (for contradiction)
- Always define p as the pumping length
- Choose s cleverly — make it depend on p
- Use $|xy| \leq p$ to locate where y can be
- Show contradiction clearly: "This is NOT in L"
- End with "Therefore, L is not regular"

MORE RESOURCES

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Other resources:
→ NFA to DFA Converter (Coming Soon)
→ Dynamic Programming Visualizer (Coming Soon)
→ Recursion Call Stack Tracer (Coming Soon)