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Pumping lemma cheat sheet

For Theory of Computation / Automata Theory
THE 5-STEP PUMPING LEMMA TEMPLATE

STEP 1: Assume L is regular.

Let p be the pumping length from the pumping lemma.
Write: "Assume L is regular. Let p be the pumping length."

STEP 2: Choose a string s ∈ L with |s| ≥ p.

The string must depend on p.
Write: "Choose s = _____ (|s| = ___ ≥ p)"

STEP 3: Consider ANY split s = xyz with |xy| ≤ p and |y| ≥ 1.

Use the condition |xy| ≤ p to determine where y can be.
Write: "For any split s = xyz with |xy| ≤ p and |y| ≥ 1..."

STEP 4: Show that $xy^0z \notin L$ OR $xy^2z \notin L$.

Pumping down (i=0) or up (i=2) creates a contradiction.
Write: "Then xy^0z = _____ ∉ L (or $xy^2z \notin L$)"

STEP 5: Contradiction → L is NOT regular.

The pumping lemma requires $xy^iz \in L$ for all i ≥ 0.
Write: "This contradicts the pumping lemma. Therefore, L is not regular."

KEY VARIABLES TO REMEMBER

p = pumping length (given by lemma)
s = chosen string (must depend on p)
x = prefix of s (|xy| ≤ p)
y = middle part (non-empty, |y| ≥ 1)
z = suffix of s
i = pumping factor (i = 0 or i = 2 for contradiction)

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Solved Examples

SOLVED EXAMPLE 1: $\{0^n1^n \mid n \geq 0\}$

Problem: Prove L = $\{0^n1^n \mid n \geq 0\}$ is not regular.
Solution:
Step 1: Assume L is regular. Let p be the pumping length.
Step 2: Choose s = 0^p1^p. Then $|s| = 2p \geq p$ and $s \in L$.
Step 3: Consider any split s = xyz with $|xy| \leq p$ and $|y| \geq 1$.
Since $|xy| \leq p$, xy contains only 0s. So $y = 0^k$ for some $k \geq 1$.
Step 4: Pump down (i = 0): $xy^0z = 0^{p-k}1^p$.
This string has fewer 0s than 1s, so it is NOT in L.
Step 5: This contradicts the pumping lemma. Therefore, L is not regular.

SOLVED EXAMPLE 2: $\{a^nb^nc^n \mid n \geq 0\}$

Problem: Prove L = $\{a^nb^nc^n \mid n \geq 0\}$ is not regular.
Solution:
Step 1: Assume L is regular. Let p be the pumping length.
Step 2: Choose s = $a^pb^pc^p$. Then $|s| = 3p \geq p$ and $s \in L$.
Step 3: Consider any split s = xyz with $|xy| \leq p$ and $|y| \geq 1$.
Since $|xy| \leq p$, xy contains only a's. So $y = a^k$ for some $k \geq 1$.
Step 4: Pump down (i = 0): $xy^0z = a^{p-k}b^pc^p$.
This string has fewer a's than b's or c's, so it is NOT in L.
Step 5: This contradicts the pumping lemma. Therefore, L is not regular.

SOLVED EXAMPLE 3: $\{ww \mid w \in \{a,b\}^*\}$

Problem: Prove L = $\{ww \mid w \in \{a,b\}^*\}$ is not regular.
Solution:
Step 1: Assume L is regular. Let p be the pumping length.
Step 2: Choose s = a^pba^pb. Then $|s| = 2p+2 \geq p$ and $s \in L$.
Step 3: Consider any split s = xyz with $|xy| \leq p$ and $|y| \geq 1$.
Since $|xy| \leq p$, y contains only a's from the first group.
Step 4: Pumping changes the number of a's in the first group only.
The resulting string cannot be of the form ww (unbalanced).
So $xy^iz \notin L$ for $i \neq 1$.
Step 5: This contradicts the pumping lemma. Therefore, L is not regular.

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COMMON MISTAKES TO AVOID

❌ MISTAKE 1: Choosing s that does NOT depend on p
WRONG: $s = 0^n1^n$
RIGHT: $s = 0^p1^p$
Why: The string must be expressed in terms of the pumping length p.

❌ MISTAKE 2: Assuming y is in a specific position
WRONG: "Let $y = 0^k$" without justification
RIGHT: "Since $|xy| \leq p$, xy contains only 0s, so $y = 0^k$"
Why: You must use the $|xy| \leq p$ condition to justify where y can be.

❌ MISTAKE 3: Forgetting the $|xy| \leq p$ condition
WRONG: Ignoring this condition entirely
RIGHT: "Because $|xy| \leq p$, we know that..."
Why: This condition is critical for determining the location of y.

❌ MISTAKE 4: Only checking i = 2 (pumping up)
WRONG: Only showing $xy^2z \notin L$
RIGHT: Consider i = 0 (pumping down) when it creates a cleaner proof
Why: Sometimes pumping down creates a simpler contradiction.

❌ MISTAKE 5: Choosing a string that doesn't allow contradiction
WRONG: Choosing $s = (ab)^p$ for language {ww}
RIGHT: Choosing $s = a^pba^pb$ for language {ww}
Why: Your chosen string must make the contradiction easy to show.

❌ MISTAKE 6: Not handling ALL possible splits
WRONG: "Assume $y = 0^k$"
RIGHT: "For any split with $|xy| \leq p$, y must consist only of 0s"
Why: The lemma says "for ANY split" — you must consider all possibilities.

PRACTICE PROBLEMS

Try to prove these languages are NOT regular using the pumping lemma.

PROBLEM 1: $L = \{a^nb^m \mid n \leq m\}$ (a's followed by b's, with fewer or equal a's)
PROBLEM 2: $L = \{0^i1^j \mid i \neq j\}$ (different number of 0s and 1s)
PROBLEM 3: $L = \{a^{b^2} \mid b \geq 0\}$ (a raised to a perfect square power)

PROBLEM 4: L = {uuv | u, v ∈ {a,b}*} (strings with two identical prefixes)
PROBLEM 5: L = {$a^n b^m$ | n = 2m} (twice as many a's as b's)

ANSWER KEY (HINTS)

PROBLEM 1: Choose s = $a^p b^p$. Since |xy| ≤ p, y contains only a's.
Pump down → fewer a's, still satisfies n ≤ m? No, contradiction.
PROBLEM 2: Choose s = $0^p 1^{p+1}$. Pumping changes count of 0s only.
Resulting string has equal counts → contradiction.
PROBLEM 3: Choose s = a^{p^2}. Pumping changes length to $p^2 + k$.
Next perfect square is $p^2 + 2p + 1$, so result not a perfect square.
PROBLEM 4: Choose s = $a^p b a^p b$. Pumping changes first group only.
String no longer has form uuv.
PROBLEM 5: Choose s = $a^{2p} b^p$. With |xy| ≤ p, y contains only a's.
Pumping changes a count, no longer twice b count.

EXAM TIPS

- Always start with "Assume L is regular" (for contradiction)
- Always define p as the pumping length
- Choose s cleverly — make it depend on p
- Use |xy| ≤ p to locate where y can be
- Show contradiction clearly: "This is NOT in L"
- End with "Therefore, L is not regular"

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